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Supposing $k \neq 0$ and dividing by $8a^2k^2$, we have

$$2 + 14ak + 15a^2k^2 = 0;$$

whence

$$ak = \frac{-7 \pm \sqrt{19}}{15}.$$

Then

$$1 + ak = \frac{8 \pm \sqrt{19}}{15}, \quad \text{and} \quad \frac{a}{1 + ak} = \frac{15a}{8 \pm \sqrt{19}}.$$

That is, if a is a positive integer, the second term of the series is irrational. Hence, the only series which satisfies the condition of the problem is one for which $k = 0$; that is, a series in which the terms are all equal.

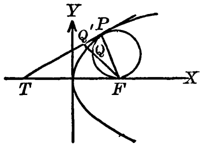
Also solved by H. L. Olson and Arthur Pelletier.

2779 [1919, 268]. Proposed by J. L. Riley, Junior Agricultural and Mechanics College, Stephensville, Texas.

A parabola is placed with its axis horizontal; find the straight line of shortest descent from the curve to the focus.

I. Solution by A. M. Harding, University of Arkansas.

Construct a circle tangent to the axis of the parabola at F and tangent to the parabola at P[1]. The time of descent down all chords of this circle which pass through F is the same, so that the time down PF is the same as the time down any other chord QF, and is, therefore, less than the time down the focal radius $Q'F$.



II. Solution by H. S. Uhler, Yale University.

Let $p \equiv$ distance from vertex to focus, $g \equiv$ acceleration due to gravity, $s \equiv$ distance from any point (x, y) on the parabola and in the first quadrant to the focus $(p, 0)$, $t \equiv$ time of descent, and $\gamma \equiv$ angle which the straight line makes with the positive direction of the axis of x. From kinematics, $s = \frac{1}{2}at^2$ and $a = g \sin \gamma$; hence,

$$t^2 = \frac{2s}{g \sin \gamma}. \tag{1}$$

The polar equation of the parabola, with the pole at the focus, is

$$s = \frac{2p}{1 - \cos \gamma} = \frac{2p(1 + \cos \gamma)}{\sin^2 \gamma}. \tag{2}$$

Hence, from (1) and (2),

$$\frac{g}{4p} t^2 = \frac{1 + \cos \gamma}{\sin^3 \gamma}.$$

Differentiating,

$$\frac{gt}{8p} \cdot \frac{dt}{d\gamma} = \frac{-\sin^2 \gamma - 3(1 + \cos \gamma) \cos \gamma}{\sin^4 \gamma} = 0;$$

therefore,

$$2 \cos^2 \gamma + 3 \cos \gamma + 1 = 0$$

so that

$$\cos \gamma = -1 \quad \text{or} \quad \cos \gamma = -\tfrac{1}{2}.$$

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[1] If this construction is possible and if the tangent at P meets the axis in T, $TP = TF = x + p$, the equation of the parabola being $y^2 = 4px$. Hence to get the coördinates of P we have $(x + p)^2 = 4x^2 + 4px$ or $x = \frac{1}{3}p$. It follows that the triangle TPF is an equilateral triangle, each side equal to $\frac{4}{3}p$. The coördinates of the center of the circle will be p, $4p/3\sqrt{3}$, and its equation $(x - p)^2 + y^2 - 8py/3\sqrt{3} = 0$. If we eliminate x or y from this equation and the equation of the parabola we shallfind for the resulting equation two equal roots corresponding to p, and two complex roots, showing that the circle must lie entirely within the parabola—which is an important consideration in connection with Professor Harding's discussion of the problem.—Editors.

An examination of the factored form of the derivative shows that $\gamma = 120°$ gives a minimum.

Incidentally,

$$t = 4\sqrt{\frac{p}{3g\sqrt{3}}}, \qquad s = \tfrac{4}{3}p.$$

Also solved by T. M. BLAKSLEE, R. A. JOHNSON, H. L. OLSON, ARTHUR PELLETIER, S. W. REAVES, J. B. REYNOLDS, and ELIJAH SWIFT.

2794 [1919 458]. Proposed by B. J. BROWN, Kansas City, Mo.

Find the value of $x^{e^x} \div x^{x^x}$ when $x \doteq 0$ and when $x \doteq \infty$. I. C. S. 1902.

SOLUTION BY PAUL CAPRON, U. S. Naval Academy.

A^{b^c} is taken to mean $A^{(b^c)}$.

(i) When $x \doteq 0$, it is well known that $e^x \doteq 1$, $x^x \doteq 1$;

$$x\,(\log x)^n \doteq \frac{(\log x)^n}{1/x} \doteq -\,n\,\frac{(\log x)^{n-1}}{1/x} \doteq \cdots \doteq \pm n!\,x \doteq 0. \qquad y = x^{e^x} \div x^{x^x} = x^{e^x - x^x}.$$

$$\log y = (e^x - x^x)\log x = \frac{e^x - x^x}{1/\log x} \doteq \frac{0}{0} \doteq \frac{e^x - x^x(1 + \log x)}{-\dfrac{1}{x(\log x)^2}}$$

$$\doteq -\,xe^x(\log x)^2 + x^x(x(\log x)^2 + x(\log x)^3 \doteq 1.0 + 1(0 + 0) = 0.$$

Hence, as $x \doteq 0$, $y \doteq 1$.

(ii) When $x \doteq \infty$,

$$x^x \div e^x = \left(\frac{x}{e}\right)^x \doteq \infty;$$

hence,

$$e^x - x^x = e^x\left(1 - \left(\frac{x}{e}\right)^x\right) \doteq -\,\infty;$$

i.e., with the notation of (i), $\log y = (e^x \dot{-} x^x)\log x \doteq (-\,\infty)(+\,\infty) \doteq -\,\infty$, or $y \doteq 0$.

2795 [1919, 458]. Proposed by C. N. SCHMALL, New York City.

A square is described touching the ellipse, $x^2/a^2 + y^2/b^2 = 1$, at the ends of its minor axis; a second ellipse is drawn circumscribing the square and tangent to the given ellipse at the ends of the major axis. The new ellipse is treated as the first and the process is continued until there are n new ellipses. Show that the last ellipse is a circle if the eccentricity of the original ellipse is $\sqrt{n/(n + 1)}$.

SOLUTION BY GERTRUDE I. MCCAIN, Oxford, Ohio.

If x', y' be the coördinates of a corner of the first square, then each equals b; and, lying on the second ellipse,

$$\frac{x'^2}{a^2} + \frac{y'^2}{b_1^2} = 1,$$

where b_1 is the minor axis of the first circumscribed ellipse. Substituting b for x' and y' and solving for b_1^2,

$$b_1^2 = \frac{a^2b^2}{a^2 - b^2}.$$

Similarly,

$$b_2^2 = \frac{a^2b_1^2}{a^2 - b_1^2} = \frac{a^2b^2}{a^2 - 2b^2},$$

and

$$b_n^2 = \frac{a^2b^2}{a^2 - nb^2}.$$